Modelling Electromagnetic Heating

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Overview

- Whitney finite elements and their properties
- Solving Maxwell' s curl equations using Whitney elements
- Treating material equations
- Results to date
- Future goals

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EM Goals

- Create a general EM solver which can be used from DC to microwave frequencies
  - Induction heating, eventually with fluid flow
  - Mold preheating
- Be able to extract resistive Joule heat generated electromagnetically and feed it back to Telluride' s thermodynamics package
- Be able to calculate Lorentz forces on any charged particles or currents and feed it back to Telluride' s flow package

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Maxwell's Equations

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

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Why Whitney Elements?

- In hyperbolic problems, such as EM, correct specification of boundaries imperative (Cauchy boundary conditions necessary)
- Scalar node elements have difficulties representing surface normals
- Vector elements needed; Whitney elements give extra benefit of making divergence and curl relations easy

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Face- and Edge- Finite Elements

- Edge elements have *circulation* 1 on an edge, circulation going linearly to 0 at adjacent edges
  - Pure circulations have no divergences
- Face elements have *flux* 1 on a face, flux going linearly to 0 at adjacent faces
  - Pure fluxes have no vorticity
- These properties solve false divergence issues
- Both are vector elements with direction; face elements give natural way to represent directed boundaries between materials

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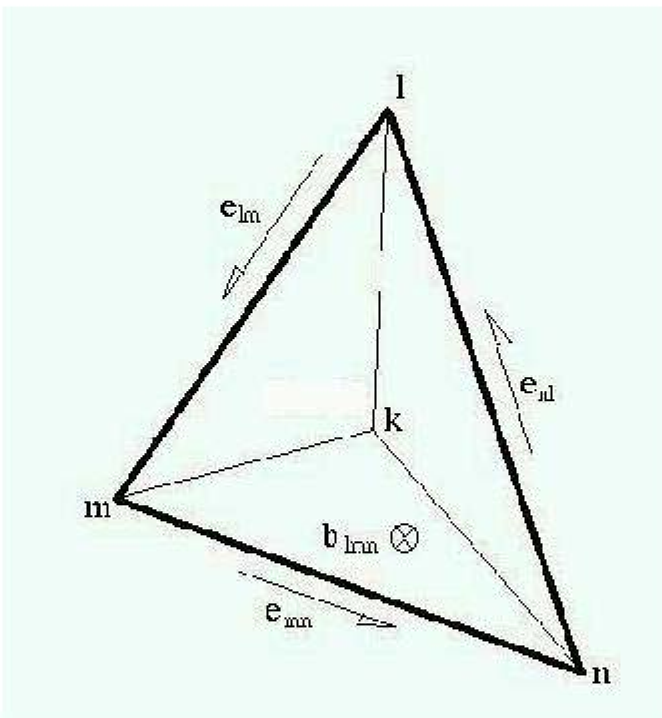
Tet with Face and Edge Elements



- We use a general tet mesh

- Shown is one element, with orientations of representative face and edge elements
(the b- flux and the
e- circulation)

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Whitney Element Setup

$$\vec{W}^{(1)}_{i,j} = \phi_i \vec{\nabla} \phi_j - \phi_j \vec{\nabla} \phi_i$$

$$\vec{W}^{(2)}_{i,j,k} = 2\phi_i (\vec{\nabla} \phi_j \times \vec{\nabla} \phi_k) + 2\phi_j (\vec{\nabla} \phi_k \times \vec{\nabla} \phi_i) + 2\phi_k (\vec{\nabla} \phi_i \times \vec{\nabla} \phi_j)$$

ϕ_i is node element associated with node i;

with value 1 at node i, going linearly to 0

at adjacent nodes, 0 everywhere else

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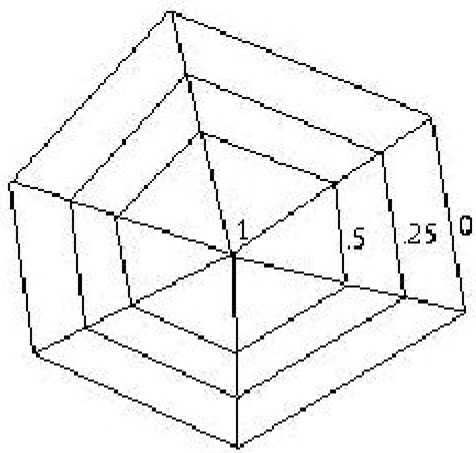
Finite Element (TD) Codes



- Uses only one grid
- Traditional node elements equal to one at grid node, going linearly to zero at all adjacent nodes, and zero thereafter (2- D linear finite element at left)

Code Construction

Let $\vec{E} = \sum_{i,j} e_{i,j} \vec{W}_{i,j}^{(1)}$

$$\vec{H} = \sum_{i,j} h_{i,j} \vec{W}_{i,j}^{(1)}$$

$$\vec{D} = \sum_{i,j,k} d_{i,j,k} \vec{W}_{i,j,k}^{(2)}$$

$$\vec{B} = \sum_{i,j,k} b_{i,j,k} \vec{W}_{i,j,k}^{(2)}$$

$$\vec{j} = \sum_{i,j,k} J_{i,j,k} \vec{W}_{i,j,k}^{(2)}$$

- $W^{(1)}$'s are edge elements with circulation 1, inde by ending, then beginning node (s over pairs of tet n
- $W^{(2)}$'s are face elements with flu indexed counter-clockwise (sums triplets of tet nod

Maxwell's Equations

$$\int d\vec{S}\cdot\dot{\vec{B}} = -\oint d\vec{l}\cdot\vec{E}$$
$$\int d\vec{S}\cdot\dot{\vec{D}} = \oint d\vec{l}\cdot\vec{H} - \int d\vec{S}\cdot\vec{J}$$

Specifying face defined by $\{l,m,n\}$, properties of face & edge elements reduce these equations to

$$\dot{b}^{l,m,n} = -(e^{l,m} + e^{m,n} + e^{n,l})$$
$$\dot{d}^{l,m,n} = h^{l,m} + h^{m,n} + h^{n,l} - J^{l,m,n}$$

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Coding Material Equations

- Maxwell's equations uncoupled; we need relations between B and H, and D and E.
- We first try to code linear materials, giving the relations: $\vec{B}=\mu\vec{H}, \quad \vec{D}=\epsilon\vec{E}$
- There are several ways to code these equations, depending on what physical or geometric principles are important to your problem
- We coded the linear material equations to preserve energy

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Energy Conservation and Materials

To find coefficients for H and E given those for B and D, Multiply both sides of material equations by an arbitrary edge element and integrate the entire volume of the mesh:

$$\sum_{i,j} h_{i,j} \int dV_{mesh} \vec{W}^{(1)}_{i,j} \cdot \vec{W}^{(1)}_{l,m} = \sum_{i,j,k} b_{i,j,k} \int \frac{dV_{mesh}}{\mu} \vec{W}^{(2)}_{i,j,k} \cdot \vec{W}^{(1)}_{l,m}$$

$$\sum_{i,j} e_{i,j} \int dV_{mesh} \vec{W}^{(1)}_{i,j} \cdot \vec{W}^{(1)}_{l,m} = \sum_{i,j,k} d_{i,j,k} \int \frac{dV_{mesh}}{\epsilon} \vec{W}^{(2)}_{i,j,k} \cdot \vec{W}^{(1)}_{l,m}$$

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Geometric Factors in Material Equations

Discretizing the mesh volume into tet cells, placing material properties inside tet cells, we define:

$$S_{i,j}^{l,m} \equiv (c\,\Delta t)^{-1} \sum_{tet\,v} \int dV_v\, \vec{W}_{i,j}^{(1)} \cdot \vec{W}_{l,m}^{(1)}$$

$$M_{i,j}^{r,s,t} \equiv \mu_0 \sum_{tet\,v} \int \frac{dV_v}{\mu_v} \vec{W}_{i,j}^{(1)} \cdot \vec{W}_{r,s,t}^{(2)}$$

$$P_{i,j}^{r,s,t} \equiv \epsilon_0 \sum_{tet\,v} \int \frac{dV_v}{\epsilon_v} \vec{W}_{i,j}^{(1)} \cdot \vec{W}_{r,s,t}^{(2)}$$

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Material Equations & Energy Conservation

With matrices defined on last slide, we have:

$$\vec{\vec{S}}(\mu_0 c \Delta t \vec{h}) = \vec{\vec{M}} \vec{b}; \quad \vec{\vec{S}}(\Delta t \vec{e}) = \vec{\vec{P}}(\mu_0 c \vec{d})$$

where vectors are vectors of expansion coefficients.

Premultiplying by circulation expansion coefficients:

$$\vec{h}^T \vec{\vec{S}}(\mu_0 c \Delta t \vec{h}) = \mu_0 \int dV_{mesh} H^2 = \vec{h}^T \vec{\vec{M}} \vec{b} = \mu_0 \int dV_{mesh} \frac{\vec{B} \cdot \vec{H}}{\mu}$$

$$\vec{e}^T \vec{\vec{S}}(\Delta t \vec{e}) = 1/c \int dV_{mesh} E^2 = \vec{e}^T \vec{\vec{P}}(\mu_0 c \vec{d}) = 1/c \int dV_{mesh} \frac{\vec{D} \cdot \vec{E}}{\epsilon}$$

shows energy is conserved across this projection.

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Properties of Equations

Combining material eqns. coded as

$$\frac{d}{dt}\vec{Y} = -\alpha\vec{Y}$$

$$\vec{Y} = \left[\vec{\vec{S}}(\mu_0 c\,\Delta t\,\vec{h}) - \vec{\vec{M}}\,\vec{b}\right]^2 \text{ or } \left[\vec{\vec{S}}(\Delta t\,\vec{e}) - \vec{\vec{P}}(\mu_0 c\,\vec{d})\right]^2$$

with Maxwell's curl eqns. where $\vec{\vec{C}}$ is discrete curl:

$$\dot{\vec{b}} = -\vec{\vec{C}}\,\vec{e} \qquad \dot{\vec{d}} = \vec{\vec{C}}\,\vec{h} - \vec{J}$$

makes a complete system of equations to be coded.

* S is N_{edges} squared large - invertable
* M and P are $N_{faces} \times N_{edges}$ large - not square or invertable
* C is $N_{edges} \times N_{faces}$ large - not square or invertable
* Time dependence only appears in Maxwell's eqns.

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Time- Stepping in Code

Theta method timestepping:

$$\Delta t\,\dot{x} \Rightarrow x_{n+1} - x_n$$

$$x \Rightarrow \vartheta\, x_{n+1} + (1-\vartheta)\, x_n$$

·ϑ is a parameter which measures the amount of implicitness or explicitness in the code.

·$\vartheta=1$ is fully implicit. $\vartheta=0$ is fully explicit.

·$\vartheta=1/2$ is the Crank-Nicolson or trapezium method.

This has no amplitude error and conserves energy.

Results and Issues

- Currently use GMRes to solve system after discretizing differential equations with theta method.
- Setting $E_z = 1$ and $B_z = 1$ initially, we try to see if code keeps these static fields stable.
- Preliminary tests show no instabilities down to 1 ppm and we can achieve timesteps of hundreds of Courant conditions on 15352- element cylindrical mesh with $\alpha = 1$.

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Second- order Solver

Checking if positive definite matrices affect stability, combine both Maxwell eqns. with both material eqns. to get two second-order DE's in one field:

$$c\,\Delta t\,\vec{\vec{S}}\,\ddot{\vec{e}}+\frac{c}{\Delta t}\vec{\vec{L}}\,\vec{e}=\frac{-1}{\epsilon}\vec{\vec{P}}\,\dot{\vec{j}}$$

$$c\,\Delta t\,\vec{\vec{S}}\,\ddot{\vec{h}}+\frac{c}{\Delta t}\vec{\vec{L}}\,\vec{h}=\frac{c}{4\,\Delta t}\vec{\vec{L}}\,\vec{\vec{C}}^{T}\,\dot{\vec{j}}$$

$$L_{e}^{e'}\equiv\frac{\Delta t}{c}\sum_{tet\,v}\int\frac{dV_{v}}{\mu_{v}\epsilon_{v}}curl\,\vec{W}_{e}^{(1)}\cdot curl\,\vec{W}_{e'}^{(1)}$$

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2' nd Order Code Setup

For an arbitrary vector of circulation coefficients $\vec{c}$ and an arbitrary vector of circulation sources $\vec{s}$ setting we can reduce the 2'nd order DE:

$$(\Delta t)^2 \vec{\vec{S}}\,\ddot{\vec{c}} + \vec{\vec{L}}\,\vec{c} = \vec{s}$$

to a system of 1'st order DE's by setting $\vec{v} \equiv \dot{\vec{c}}$

$$\begin{bmatrix} \vec{\vec{I}} & 0 \\ 0 & \Delta t\,\vec{\vec{S}} \end{bmatrix} \begin{bmatrix} \dot{\vec{c}} \\ \Delta t\,\dot{\vec{v}} \end{bmatrix} = \begin{bmatrix} 0 & (\Delta t)^{-1}\,\vec{\vec{I}} \\ -\vec{\vec{L}} & 0 \end{bmatrix} \begin{bmatrix} \vec{c} \\ \Delta t\,\vec{v} \end{bmatrix} + \begin{bmatrix} 0 \\ \vec{s} \end{bmatrix}$$

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2' nd- Order Solver Results

- We apply the theta method to this differential matrix equation, invert the matrix on the left-hand side and run the static field problem on this field
- No instability on the 72- element mesh and, we can achieve timesteps of hundreds of Courant conditions on 15352- element mesh.
- Unusual stability of $\theta = 0$, boundary conditions less transparent, and matrices twice as large as first- order code.

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Future Directions

- More complex tests - waves and magnetic dipoles
  - Waves need resistance, boundary conditions, and packet construction on unstructured meshes
  - Dipoles need fields generated solely from currents
- Add resistance via Ohm's Law ($j = \sigma E$) to give electromagnetic energy lost (σE^2)
- Integration with rest of truchas, Parallelization, static electric charges & fields, non- tet meshes ...

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Finite element setup

For basis on element:

$$\phi_{e,i} = (\alpha_{e,i} + \beta_{e,i}\, x + \delta_{e,i}\, y + \zeta_{e,i}\, z)/6\, V_e$$

Let $\boldsymbol{A} = \begin{bmatrix} 1 & x_{e,1} & y_{e,1} & z_{e,1} \\ 1 & x_{e,2} & y_{e,2} & z_{e,2} \\ 1 & x_{e,3} & y_{e,3} & z_{e,3} \\ 1 & x_{e,4} & y_{e,4} & z_{e,4} \end{bmatrix}$ then:

$$\alpha_{e,i} = cof(A_{i,1}),\ \ \beta_{e,i} = cof(A_{i,2}),\ \ \delta_{e,i} = cof(A_{i,3})$$

$$\zeta_{e,i} = cof(A_{i,4}),\ \ V_e = det(\boldsymbol{A})/6$$

node 1, element e at $(x_{e,1}, y_{e,1}, z_{e,1})$, etc.

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Two- Field Solver

Premultiply the Maxwell curl equations by $\vec{\vec{M}}$ and $\vec{\vec{P}}$:

$$\vec{\vec{M}}\,\vec{b} = \vec{\vec{S}}\left(\mu_0 c\, \Delta t\, \dot{\vec{h}}\right) = -\vec{\vec{M}}\,\vec{\vec{C}}\,\vec{e}$$

$$\vec{\vec{P}}\left(\mu_0 c\, \dot{\vec{d}}\right) = \vec{\vec{S}}\left(\Delta t\, \dot{\vec{e}}\right) = \vec{\vec{P}}\,\vec{\vec{C}}\left(\mu_0 c\, \vec{h}\right) - \mu_0 c\, \vec{\vec{P}}\,\vec{J}$$

In matrix form, and applying theta method gives:

$$\begin{bmatrix} \vec{\vec{S}} & -\vartheta\,\vec{\vec{P}}\,\vec{\vec{C}} \\ \vartheta\,\vec{\vec{M}}\,\vec{\vec{C}} & \vec{\vec{S}} \end{bmatrix} \begin{bmatrix} \vec{e}_{n+1} \\ \mu_0 c\, \vec{h}_{n+1} \end{bmatrix} =$$

$$\begin{bmatrix} \vec{\vec{S}} & (1-\vartheta)\,\vec{\vec{P}}\,\vec{\vec{C}} \\ (\vartheta-1)\,\vec{\vec{M}}\,\vec{\vec{C}} & \vec{\vec{S}} \end{bmatrix} \begin{bmatrix} \vec{e}_{n} \\ \mu_0 c\, \vec{h}_{n} \end{bmatrix} + \mu_0 c \begin{bmatrix} \vec{\vec{P}}\,\tilde{\vec{j}} \\ 0 \end{bmatrix}$$

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Results and Issues w/2- Field Solver

- Once again, we invert the matrix on the left- hand side, and try to keep static fields constant.
- Unstable for 72- element cube mesh, better results for 15352- element cylinder mesh.
- Can get up to about 10 Courant conditions quickly and stably for implicit cases and $\theta = 1$, 2/3 of this for $\theta = ½$
- We can show MC and PC are symmetric with zero trace - not positive definite

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Treating Linear Materials

$$\vec{B}=\mu\vec{H},\quad \vec{D}=\epsilon\vec{E},\quad \text{become}$$

$$\sum_{i,j} h^{i,j}\vec{W}^{(1)}_{i,j}=\sum_{i,j,k}(b^{i,j,k}/\mu)\vec{W}^{(2)}_{i,j,k}$$

$$\sum_{i,j} e^{i,j}\vec{W}^{(1)}_{i,j}=\sum_{i,j,k}(d^{i,j,k}/\epsilon)\vec{W}^{(2)}_{i,j,k}$$

Integrating across edge $\{m,n\}$:

$$h^{m,n}=\sum_{i,j,k}(b^{i,j,k}/\mu)\int d\vec{l}_{m,n}\cdot\vec{W}^{(2)}_{i,j,k}$$

$$e^{m,n}=\sum_{i,j,k}(d^{i,j,k}/\epsilon)\int d\vec{l}_{m,n}\cdot\vec{W}^{(2)}_{i,j,k}$$

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